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FUZZY ANALYTICAL HIERARCHY PROCESS FOR MULTICRITERIA INVENTORY CLASSIFICATION

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ABSTRACT

Inventory has been looked into as a major cost and source of uncertainty due to the volatility within the commodity market and demand for the value-added product. Because of the huge number of inventory items in many companies, great attention is directed to inventory classification into the different classes, which consequently require the application of different management tools and policies. Sometimes, only one criterion is not a very efficient measure for decision-making. Therefore, multiple criteria decision making methods are used. In this paper, fuzzy analytic hierarchy process for multiple criteria ABC inventory classification has been proposed. Fuzzy Analytic Hierarchy Process (Fuzzy AHP) is used to determine the relative weights of the attributes or criteria, and to classify inventories into different categories. To accredit the proposed model, it is implemented for the 351 raw materials of switch gear section of Energypac Engineering Limited (EEL), power engineering company of Bangladesh.

Keywords: Multicriteria Inventory Classification, Fuzzy AHP, Triangular Fuzzy Number.

1. INTRODUCTION

Inventory has been looked at as a major cost and source of uncertainty due to the volatility within the commodity market and demand for the value-added product. ABC inventory management deals with classification of the items in an inventory in decreasing order of annual dollar volume. The ABC classification process is an analysis of a range of items, such as finished products or customers into three categories: A - outstandingly important; B - of average importance; C - relatively unimportant as a basis for a control scheme. Each category can be and sometimes should be handled in a different way, with more attention being devoted to category A, less to B, and less to C [1].

Sometimes, only one criterion is not a very efficient measure for decision-making. Therefore, multiple criteria decision making methods are used [2-3]. Apart from other criteria like lead time of supply, part criticality, availability, stock out penalty costs, ordering cost, scarcity, durability, substitutability, reparability etc has been taken into consideration [2-16]. More studies have been done on multi-criteria inventory classification in the past 20 years. So many different methods for classifying inventory and taking into consideration multiple criteria have been used and developed.

Flores and Whybark [2-3] proposed the bi-criteria matrix approach, wherein annual dollar usage by a joint-criteria matrix is combined with another criterion. Flores et al. [4] have proposed the use of joint criteria matrix for two criteria. Partovi and Burton [5] applied the analytic hierarchy process (AHP) to inventory classification in order to include both quantitative and qualitative evaluation criteria. Guvenir and Erel [6] applied genetic algorithm technique to the problem of multiple criteria inventory classification. Partovi and Anandarajan [7] proposed an artificial neural network (ANN) approach for inventory classification. Braglia et al. [8] integrated decision diagram with a set of analytic hierarchy process (AHP) models used to solve the various multi-attribute decision sub-problems at the different levels/nodes of the decision tree. Lei et al. [9] compared principal component analysis with a hybrid model combining principal component analysis with artificial neural network and back propagation algorithm. Liu and Huang [10] present a modified Data Envelopment Analysis (DEA) model to address ABC inventory classification. Bhattacharya et al. [11] developed a distance-based multiple-criteria consensus framework utilizing the technique for order preference by similarity to ideal solution (TOPSIS) for ABC analysis. Chen et al. [12] proposed a case-based distance model for multiple criteria ABC analysis. Jamshidi and Jain [13] addressed multi-criteria ABC inventory classification to standardized each criterion and weight them for classification. Šimunović et al. [14] investigated the application of neural networks in multiple criteria inventory classification. Hadi-Vencheh [15] proposed a simple nonlinear programming model which determines a common set of weights for all the items. Yu [16] compared artificial-intelligence (AI)-based classification techniques with traditional multiple discriminant analysis (MDA). Therefore the main objective of this research is to develop an improved multi-criteria inventory classification model using Fuzzy

Analytic Hierarchy Process (FAHP) approach.

2. FUZZY ANALYTIC HIERARCHY PROCESS

There are several methods to determine of the criteria weights, including analytic hierarchy process (AHP), entropy analysis, eigenvector method, weighted least square method and linear programming for multidimensions of analysis preference (LINMAP). In this model, the method of fuzzy analytic hierarchy process (FAHP) is applied. There are the several procedures to attain the priorities in FAHP. The fuzzy least square method [17], method based on the fuzzy modification of the LLSM [18], geometric mean method [19], the direct fuzzification of the method of Csutora and Buckley [20], synthetic extend analysis [21], Mikhailov's fuzzy preference programming [22] and two-stage logarithmic programming [23] are some of these methods. Chang's extent analysis is utilized in this research to evaluate the focusing problem.

Chang [24] introduces a new approach for handling pair-wise comparison scale based on triangular fuzzy numbers followed by use of extent analysis method for synthetic extent value of the pairwise comparison [21]. The first step in this method is to use triangular fuzzy numbers for pairwise comparison by means of FAHP scale, and the next step is to use extent analysis method to obtain priority weights by using synthetic extent values. The fuzzy evaluation matrix of the criteria was constructed through the pairwise comparison of different attributes relevant to the overall objective using the linguistic variables and triangular fuzzy numbers (Table 1 and Figure 1).

Table 1: Linguistic variables describing weights of the criteria and values of ratings

Linguistic scale for importance	Fuzzy numbers	Membership function	Triangular fuzzy scale (*l, m, u*)
Just equal			(1, 1, 1)
Equally important	$\tilde{1}$	$\mu_M(x) = (3-x)/(3-1)$	(1, 1, 3)
Weakly important	$\tilde{3}$	$\mu_M(x) = (x-1)/(3-1)$ $\mu_M(x) = (5-x)/(5-3)$	(1, 3, 5)
Essential or Strongly important	$\tilde{5}$	$\mu_M(x) = (x-3)/(5-3)$ $\mu_M(x) = (7-x)/(7-5)$	(3, 5, 7)
Very strongly important	$\tilde{7}$	$\mu_M(x) = (x-5)/(7-5)$ $\mu_M(x) = (9-x)/(9-7)$	(5, 7, 9)
Extremely Preferred	$\tilde{9}$	$\mu_M(x) = (x-7)/(9-7)$	(7, 9, 9)
If factor *i* has one of the above numbers assigned to it when compared to factor *j*, then *j* has the reciprocal value when compare to *i*			Reciprocals of above $\tilde{M}_1^{-1} = (1/u_1, 1/m_1, 1/l_1)$

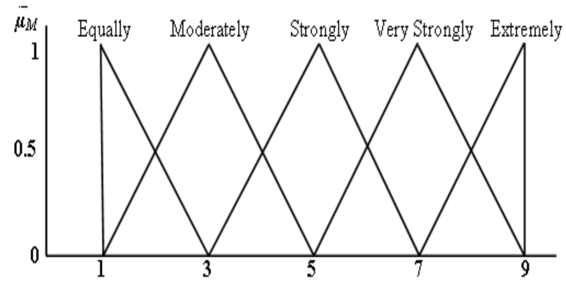

Fig 1. Linguistic variables for the importance weight of each criterion

The following section outlines the Chang's extent analysis method on FAHP. Let $X = \{x_1, x_2, \ldots, x_n\}$ be an object set and $U = \{u_1, u_2, \ldots, u_m\}$ be a goal set. As per Chang [21, 24] each object is taken and analysis for each goal, g_i, is performed, respectively. Therefore *m* extent analysis values for each object can be obtained, as under:

$M_{g_i}^1, M_{g_i}^2, \ldots, M_{g_i}^m$, $i = 1, 2, 3, \ldots, n$

where all the $M_{g_i}^m$ ($j = 1, 2, \ldots, m$) are TFNs whose parameters are, depicting least, most and largest possible values respectively and represented as (*a*, *b*, *c*).

The steps of Chang's extent analysis [21] can be detailed as follows [25-28]:

Step 1: The value of fuzzy synthetic extent with respect to *i* th object is defined as

$$S_i = \sum_{j=1}^{m} M_{g_i}^j \otimes \left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^j\right]^{-1}$$

To obtain $\sum_{j=1}^{m} M_{g_i}^j$ perform the fuzzy addition operation of *m* extent analysis values for a particular matrix such that

$$\sum_{j=1}^{m} M_{g_i}^j = \left(\sum_{j=1}^{m} a_j, \sum_{j=1}^{m} b_j, \sum_{j=1}^{m} c_j\right)$$

And to obtain $\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^j\right]^{-1}$ perform the fuzzy addition operation of $M_{g_i}^m$ ($j = 1, 2, \ldots, m$) values such that

$$\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^j = \left(\sum_{i=1}^{n} a_i, \sum_{i=1}^{n} b_i, \sum_{i=1}^{n} c_i\right)$$

And then compute the inverse of the vector in equation (11) such that

$$\left[\sum_{i=1}^{n}\sum_{j=1}^{m} M_{g_i}^j\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} c_i}, \frac{1}{\sum_{i=1}^{n} b_i}, \frac{1}{\sum_{i=1}^{n} a_i}\right)$$

Step 2: The degree of possibility of $M_2 = (a_2, b_2, c_2) \geq M_1 = (a_1, b_1, c_1)$ is defined as

$V(M_2 \geq M_1) = \sup [\min (\mu_{M_1}(x), \mu_{M_2}(x))]$

And can be equivalently expressed as follows:

$$V(\tilde{M}_2 \geq \tilde{M}_1) = hgt(\tilde{M}_1 \cap \tilde{M}_2)$$

$$= \begin{cases} 1, & \text{if } b_2 \geq b_1 \\ 0, & \text{if } a_1 \geq c_2 \\ \dfrac{a_1 - c_2}{(b_2 - c_2) - (b_1 - a_1)}, & \text{otherwise} \end{cases}$$

where *d* is the ordinate of the highest intersection point *D* between $\boldsymbol{\mu_{M_1}}$ and $\boldsymbol{\mu_{M_2}}$ as shown in Figure 2.

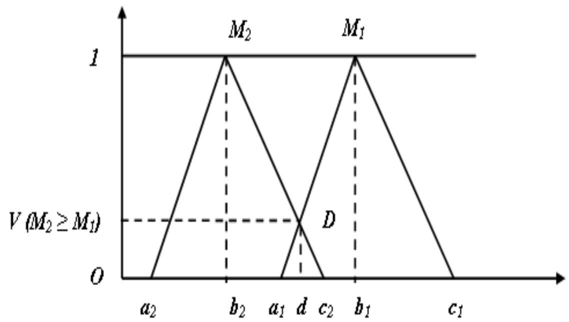

Fig 2. The intersection between M_1 and M_2

To compare M_1 and M_2, both the values of $V(M_1 \geq M_2)$ and $V(M_2 \geq M_1)$.

Step 3: The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers M_i (i = 1,2,….., k) can be defined by

$V(M \geq M_1, M_2, \ldots, M_k) = V[(M \geq M_1) \text{ and } (M \geq M_2) \text{ and } \ldots (M \geq M_k)] = \min V(M \geq M_i)$, ($i$ = 1, 2, 3 ,…., k)

Assuming that
$d'(A_i) = \min V(S_i \geq S_k)$
for $k = 1, 2, 3, \ldots, n;\ k \neq i$. Then the weight vector is given by

$W' = (d'(A_1), d'(A_2), \ldots, d'(A_n))^T$
where $A_i = (i = 1,2,3,\ldots n)$ are n elements

Step 4: By normalizing, the normalized weight vectors are
$W = (d(A_1), d(A_2), \ldots, d(A_n))^T$
where W is a non-fuzzy number.

3. APPLICATION OF THE MODEL

To accredit the proposed model, it is implemented for the 351 raw materials of switch gear section of Energypac Engineering Limited (EEL), one of the leading power engineering companies in Bangladesh. Fuzzy Analytic Hierarchy Process (Fuzzy AHP) is used to determine the relative weights of the attributes or criterions and to classify inventories into different categories through training the data set.

3.1 Determination of Criteria

Based on the extensive literature review, experts participating in the implementation of this model have regarded five important criteria for classification of inventory. Those are: Unit Price, Annual Demand, Criticality, Last Use Date and Durability.

3.2 Determination of the Weights of Criteria Using FAHP

For Multicriteria inventory classification, a questionnaire was designed to elicit judgments about the relative importance of each of the selected criteria. The questionnaire was completed by fourteen experts, among them three academia’s and eleven professional including raw material and inventory manager of EEL. Table 3 shows the aggregated fuzzy pairwise comparisons of the fourteen experts or decision maker’s. The aggregated decision matrix as shown in Table 2 is constructed to measure the relative degree of importance for each criterion, based on the Chang’s extent analysis.

Inconsistency of TFN used can be checked and the consistency ratio (CR) has to calculate. The results obtained are: largest eigen value of matrix, λ_{max} = 5.323; Consistency Index (C.I.) = 0.08075; Randomly Generated Consistency Index (R.I.) = 1.12 and Consistency Ratio (C.R.) = 0.0721 As CR < 0.1 the level of inconsistency present in the information stored in comparison matrix is satisfactory [29].

S_U = (4.16, 6.51, 11.08) ⊗ (1/42.14, 1/27.68, 1/19.13) = (0.09, 0.235, 0.58)
S_A = (5.43, 8.06, 12.76) ⊗ (1/42.14, 1/27.68, 1/19.13) = (0.13, 0.291, 0.67)
S_C = (3.23, 4.38, 6.41) ⊗ (1/42.14, 1/27.68, 1/19.13) = (0.077, 0.158, 0.34)
S_L = (3.4, 4.51, 6.19) ⊗ (1/42.14, 1/27.68, 1/19.13) = (0.08, 0.163, 0.32)
S_D = (2.91, 4.23, 5.7) ⊗ (1/42.14, 1/27.68, 1/19.13) = (0.07, 0.153, 0.30)

The degree of possibility of superiority of S_U is calculated and is denoted by $V(S_U \geq S_A)$. Therefore, the degree of possibility of superiority for the first requirement- the values are calculated as
$V(S_U \geq S_A) = 0.9$, $V(S_U \geq S_C) = 1$,
$V(S_U \geq SL) = 1$, $V(S_U \geq S_D) = 1$,
For the second requirement- the values are calculated as
$V(S_A \geq S_U) = 1$, $V(S_A \geq S_C) = 1$,
$V(S_A \geq S_L) = 1$, $V(S_A \geq S_D) = 1$,
For the third requirement- the values are calculated as
$V(S_C \geq S_U) = 0.75$, $V(S_C \geq S_A) = 0.61$,
$V(S_C \geq S_L) = 0.98$, $V(S_C \geq S_D) = 1$,
For the fourth requirement- the values are calculated as
$V(S_L \geq S_U) = 0.75$, $V(S_L \geq S_A) = 0.60$,
$V(S_L \geq S_C) = 1$, $V(S_L \geq S_D) = 1$,
For the fifth requirement- the values are calculated as
$V(S_D \geq S_U) = 0.70$, $V(S_D \geq S_A) = 0.55$,
$V(S_D \geq S_C) = 0.98$, $V(S_D \geq S_L) = 0.96$,

The minimum degree of possibility of superiority of each criterion over another is obtained. This further decides the weight vectors of the criteria. Therefore, the weight vector is given as
W' = (0.9, 1, 0.61, 0.60, 0.55)

The normalized value of this vector decides the priority weights of each criterion over another. The normalized weight vectors are calculated as
W = (0.246, 0.273, 0.167, 0.164, 0.15)

The normalized weight of each success factor is depicted in Figure 3. Figure 3 show that the annual demand has higher priority than the other criteria. The weights of the criteria represent the ratio of how much more important one criterion is than another, with respect to the goal or criterion at a higher level.

Table 2: Aggregated fuzzy comparison matrix of the attributes with respect to the overall objective

Attributes	Unit Price	Annual Demand	Criticality	Last Use Date	Durability
Unit Price	1,1,1	0.89,1.6,2.25	0.65,1.07,1.88	0.82,1.47,2.76	0.8,1.37,3.19
Annual Demand	0.44,0.62,1.12	1,1,1	2.02,3.08,4.64	0.80,1,1.47	1.17,2.36,4.53
Criticality	0.53,0.93,1.53	0.22,0.34,0.50	1,1,1	0.68,1.11,1.66	0.80,1,1.72
Last Use Date	0.36,0.68,1.21	0.68,1,1.26	0.60,0.90,1.47	1,1,1	0.76,0.93,1.25
Durability	0.31,0.73,1.26	0.22,0.42,0.86	0.58,1,1.26	0.80,1.08,1.32	1,1,1

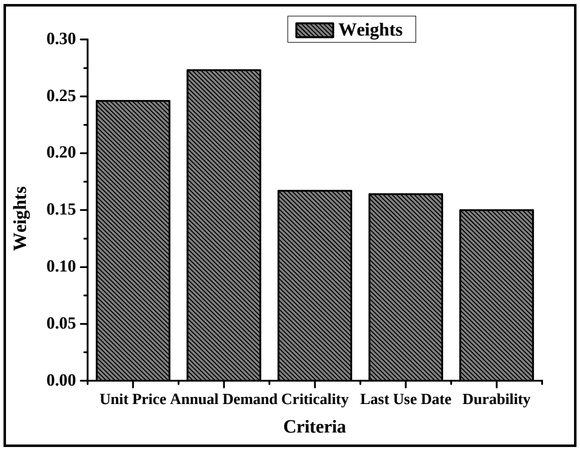

Fig 3. Normalized weights of criteria for multiple criteria inventory classification

3.3 Data Collection

Unit price, last year consumption or annual demand, last use date, criticality, durability of 351 materials of switch gear section has been collected. Range and value for the transformation of last use date, criticality and durability are shown in Table 3-5.

Table 3: Transformation of last use date

Range	Value
Used within a day	10
Used within a week	8
Used within a month	6
Used within 6 month	4
Used within a year	2
Used more than a year	1

Table 4: Transformation of criticality

Range	Value
Extremely Critical	5
Moderate Critical	3
Non Critical	1

Table 5: Transformation of durability

Mean Time Between Failure	Value
> 1 Week	10
> 1 Month	8
> 6 Month	5
> 1 year	3
< 1 year	1

3.4 Determination of Composite Priority Weights

In FAHP methodology, for a very large number of alternatives (351), making pair wise comparisons of alternatives, with respect to each criterion, can be time consuming and confusing, because the total number of comparisons will also be very big. Therefore, multiple criteria inventory classification is carried out by using the modified FAHP methodology, which includes pair wise comparisons of criteria, but not pair wise comparisons of alternatives. Because of the large number of alternatives (351), pair wise comparisons of the alternatives are not performed. Finally, the composite priority weights of each alternative have been calculated. Items are ranked according to overall composite priority weights in the descending order. Class A, B and C involve 10%, 20% and 70% of the total composite priority weights respectively. The results of the study show that among 351 items 22 items are identified as class A or very important group or outstandingly important, 47 items as class B or important group or average important and the remaining 282 items as class C or unimportant group or relatively unimportant as a basis for a control scheme.

4. DISCUSSIONS

Fuzzy linguistic terms has been employed for facilitating the comparisons between the subject criteria, since the decision makers feel much comfortable with using linguistic terms rather than providing exact crisp judgments. Using Chang's extent analysis, the normalized weight of each attributes is depicted which in shown in Figure 3. Figure 3 show that the annual demand has higher priority (0.273) than the other criteria. The composite priority weight of each alternative has been calculated using the modified FAHP methodology. The composite priority weight of the alternatives gives the idea about the appropriate class of the alternatives or items. Class A involves 10 % of the total composite priority weights. Class B involves 20 % of the total composite priority weights amount of items, while 70 % of total composite priority weights belong to class C. The results of the study show that among 351 items 22 items are identified as class A or very important group or outstandingly important, 47 items as class B or important group or average important and the remaining 282 items as class C or unimportant group or relatively unimportant as a basis for a control scheme.

5. CONCLUSIONS

In today's manufacturing and business environment, an organization must maintain an appropriate balance between critical stock-outs and inventory holding costs. Because customer service is not a principal factor for attracting new customers, but it is frequently a major reason for losing them. Many researchers have devoted themselves to achieving this appropriate balance.

In this research, a new multi-criteria inventory classification model has been proposed using Fuzzy Analytic Hierarchy Process (FAHP) approach. Fuzzy AHP technique was used to synthesize the opinions of the decision makers to identify the weight of each criterion. The FAHP approach proved to be a convenient method in tackling practical multi-criteria decision making problems. It demonstrated the advantage of being able to capture the vagueness of human thinking and to aid in solving the research problem through a structured manner and a simple process. Further development of FAHP application could be the improvement in the determination of the weights of each component and to handle uncertainty level of the decision environment by using hybrid neuro-fuzzy models, like the quick fuzzy backpropagation algorithm.

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